Mirror symmetry for Higgs bundles II

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BRANES

- symplectic geometry:

  A-brane = Lagrangian submanifold + flat connection

- complex geometry:

  B-brane = complex submaniifold + holomorphic bundle

- .... and generalizations, sheaves etc.

- hyperkähler: complex structures I, J, K

- symplectic forms $\omega_1, \omega_2, \omega_3$

- BAA-brane = holomorphic Lagrangian submanifold wrt I + flat connection

- BBB-brane = hyperkähler submanifold + hyperholomorphic bundle

- hyperkähler: complex structures I, J, K
- symplectic forms $\omega_1, \omega_2, \omega_3$
- BAA-brane = holomorphic Lagrangian submanifold wrt I + flat connection
- BBB-brane = hyperkähler submanifold + hyperholomorphic bundle

mirror symmetry is supposed to interchange these

- SYZ for hyperkähler:

$$TM \cong T_F \oplus p^*TB$$

$$K = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad J = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

- $V \subset p^*T_B$ complex subspace

- K-invariance $\Rightarrow V \oplus V \subset T_F \oplus p^*T_B$ quaternionic = BBB

- SYZ for hyperkähler: mirror

$TM \cong T_F^* \oplus p^*TB$

$$K = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad J = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

- $V^0 \oplus V \subset T_F^* \oplus p^*T_B$ complex Lagrangian

- Lagrangian wrt ω_1, ω_3, complex wrt $J =$ ABA

- semi-flat

- $V \subset B$ affine linear complex subspace

- $\Rightarrow$ induced special Kähler structure

- $\Rightarrow$ semi-flat hyperkähler submanifold

BBB-BRANES

- hyperkähler submanifold
- strong condition
- Kähler $\Rightarrow$ second fundamental form S complex linear

  $S(IX, JY) = IS(X, JY) = IJS(X, Y) = JIS(X, Y) = 0$
- $\Rightarrow$ totally geodesic

EXAMPLES FOR HIGGS BUNDLES

- moduli space for a subgroup $H \subset GL(n, \mathbf{C})$

- holomorphic automorphism of $\Sigma \Rightarrow$ fixed-point set on $\mathcal{M}$ hyperkähler

- fixed-point set of $(V, \Phi) \mapsto (V \otimes L, \Phi)$ where L^m is trivial

BAA BRANES

- $\mathcal{N} \subset \mathcal{M}$ = moduli space of stable bundles $(V, 0)$
- torus fibres of the integrable system
- $T^*\mathcal{N} \subset \mathcal{M}$ open embedding ....
- .... closure of conormal bundle of submanifold of $\mathcal{N}$

- BBB-brane = hyperkähler submanifold

  + hyperholomorphic bundle

  (support could be $\mathcal{M}$ itself)

- hyperholomorphic = connection whose curvature is of type $(1,1)$ wrt I, J, K

- e.g. instantons on $\mathbb{C}^2$ or K3

- e.g. tangent bundle of a hyperkähler manifold

SYZ MIRROR SYMMETRY

- $p : \mathcal{M} \to B$ Lagrangian fibration

  $Y \subset \mathcal{M}$ Lagrangian submanifold

- $b \in p(Y) \subset B$ suppose $p^{-1}(b) = A$ smooth fibre

- **Define** $(A \cap Y)^0 \subset A^\vee =$ line bundles on A trivial on Y

- .... $=$ support of BBB-brane ?

EXAMPLE 1

- take Lagrangian submanifold $Y \subset \mathcal{M}$ = smooth fibre
- $p(Y)$ = point b, $Y \cap A = A \Rightarrow (Y \cap A)^0 = 0 \in A^\vee$
- $\Rightarrow$ BBB-brane is a single point

EXAMPLE 2

- Lagrangian submanifold $Y \subset \mathcal{M}$ = cotangent fibre $T^*_x \subset T^*\mathcal{N}$
- $Y \cap A$ is generically finite $\Rightarrow p(Y) = B$
- .. and "annihilator" $(A \cap Y)^0 = A^\vee$
- $\Rightarrow$ BBB-brane is full space $\mathcal{M}^\vee = \mathcal{M}(^L G)$

IN GENERAL..?

- any morphism $X \to A$ factors through the Albanese
- and $\mathsf{Alb}(X)^\vee = \mathsf{Pic}^0(X)$
- $\Rightarrow (A \cap Y)^0 = (\mathsf{Alb}(A \cap Y))^0$

IN GENERAL..?

- any morphism $X \to A$ factors through the Albanese
- and $\mathrm{Alb}(X)^\vee = \mathrm{Pic}^0(X)$
- $\Rightarrow (A \cap Y)^0 = (\mathrm{Alb}(A \cap Y))^0$
- invert and the BAA-brane is smaller unless $Y \cap A$ is an abelian subvariety.

REAL FORMS

- complex structure I: moduli space of (stable) pairs (A, Φ)

  $G = U(n)$ vector bundle V, $\Phi \in H^0(\Sigma, \mathrm{End}\, V \otimes K)$

- complex structure J: flat G^c-connection

  $\nabla_A + \Phi + \Phi^*$ (representations $\pi_1(\Sigma) \to G^c$)

- complex structure K: flat G^c-connection

  $\nabla_A + i\Phi - i\Phi^*$

REAL FORM G^r

- $K \subset G^r$ maximal compact
- principal K^c-bundle
- $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{m}$
- Higgs field $\Phi \in H^0(\Sigma, \mathfrak{m} \otimes K)$
- holonomy of $\nabla + \Phi + \Phi^* \in G^r$

- moduli space of flat G^r-connections: $\mathrm{Hom}(\pi_1, G^r)/G^r$

- fixed point set of involution on $\mathcal{M}$

- I-holomorphic, J, K-antiholomorphic

- BAA-brane

SPLIT REAL FORM

- e.g. $SL(n, \mathbf{R}) \subset SL(n, \mathbf{C}), Sp(2m, \mathbf{R}) \subset Sp(2m, \mathbf{C})$...

- Hom$(\pi_1, G^r)/G^r$ intersects a generic fibre in 2-torsion points

- BBB-brane supported on whole space

$U(m,m) \subset GL(2m, \mathbf{C})$

- maximal compact $U(m) \times U(m)$
- bundle $V = V_+ \oplus V_-$ Higgs field $\Phi = \begin{pmatrix} 0 & \beta \\ \gamma & 0 \end{pmatrix}$
- characteristic class $c_1(V_+) \in H^2(\Sigma, \mathbf{Z})$
- $\Rightarrow$ different topological components

- spectral curve $\det(x - \Phi) = x^{2m} + a_1 x^{2m-2} + \ldots + a_m$

- involution $\sigma(x) = -x$ on S

- $L = U\pi^* K^{(2m-1)/2}$, $U \in \mathrm{Jac}(S)$

- $\sigma^* U \cong U$

- $Y \cap A =$ fixed point set of $\sigma =$ abelian subvariety

- $Y \cap A$ = fixed point set of σ = abelian subvariety

- $(Y \cap A)^0 \cong \{U : \sigma^* U \cong U^*\}$ = fibre for $Sp(2m, \mathbf{C})$

- BBB-brane = $\mathcal{M}(Sp(m)) \subset \mathcal{M}(U(2m))$

- ... which is a hyperkähler submanifold

S-Duality Of Boundary Conditions in $\mathcal{N} = 4$ Super Yang-Mills Theory

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Abstract

By analyzing brane configurations in detail, and extracting general lessons, we develop methods for analyzing S-duality of supersymmetric boundary conditions in $\mathcal{N} = 4$ super Yang-Mills theory. In the process, we find that S-duality of boundary conditions is closely related to mirror symmetry of three-dimensional gauge theories, and we analyze the IR behavior of large classes of quiver gauge theories.

Table 3: The first column lists the unbroken subgroups H in boundary conditions in $SU(n)$ gauge theory that are defined by an involution τ. The second column lists the unbroken gauge symmetry $\widetilde{H}$ of the S-dual boundary condition. The third column describes the Nahm pole, if any, that is part of the reduction of the dual gauge group from $SU(n)$ to $\widetilde{H}$. The fourth column describes the matter system that is coupled to $\widetilde{H}$. (The hypermultiplets indicated are in the fundamental representation of $Sp(n)$.)

H	$\widetilde{H}$	Nahm Pole	Matter System
$SO(n)$	$SU(n)$	None	Non-trivial SCFT
$Sp(n)$	$SU(n/2)_2$	$n = 2 + 2 + \cdots + 2$	None
$S(U(n/2) \times U(n/2))$	$Sp(n)$	None	Hypermultiplets
$S(U(p) \times U(q)),\ p > q$	$Sp(2q)$	$n = (p - q) + 1 + 1 + \cdots + 1$	None

real forms of G^c complex subgroups of ${}^LG^c$

	$\mathfrak{g}_\mathbb{R}$	$\mathfrak{g}$	$\check{\mathfrak{g}}$	$\check{\mathfrak{h}}$	Remarks
AI	$\mathfrak{sl}_n(\mathbb{R})$	$\mathfrak{sl}_n(\mathbb{C})$	$\mathfrak{sl}_n(\mathbb{C})$	$\mathfrak{sl}_n(\mathbb{C})$	split
AII	$\mathfrak{su}^*(2n)$	$\mathfrak{sl}_{2n}(\mathbb{C})$	$\mathfrak{sl}_{2n}(\mathbb{C})$	$\mathfrak{sl}_n(\mathbb{C})$	
AIII/AIV	$\mathfrak{su}(p,q)$	$\mathfrak{sl}_n(\mathbb{C})$	$\mathfrak{sl}_n(\mathbb{C})$	$\mathfrak{sp}_p(\mathbb{C})$	$p \leq q$
					$p+q=n$
					quasi-split if $q=p$
					or $q=p+1$
BI/BII	$\mathfrak{so}(p,q)$	$\mathfrak{so}_{2n+1}(\mathbb{C})$	$\mathfrak{sp}_n(\mathbb{C})$	$\mathfrak{sp}_p(\mathbb{C})$	$p<q$
					$p+q=2n+1$
					split if $q=p+1$
CI	$\mathfrak{sp}_n(\mathbb{R})$	$\mathfrak{sp}_n(\mathbb{C})$	$\mathfrak{so}_{2n+1}(\mathbb{C})$	$\mathfrak{so}_{2n+1}(\mathbb{C})$	split
CII	$\mathfrak{sp}(p,q)$	$\mathfrak{sp}_n(\mathbb{C})$	$\mathfrak{so}_{2n+1}(\mathbb{C})$	$\mathfrak{sp}_p(\mathbb{C})$	$p \leq q$
					$p+q=n$
DI/DII	$\mathfrak{so}(n,n)$	$\mathfrak{so}_{2n}(\mathbb{C})$	$\mathfrak{so}_{2n}(\mathbb{C})$	$\mathfrak{so}_{2n}(\mathbb{C})$	split
	$\mathfrak{so}(p,q)$	$\mathfrak{so}_{2n}(\mathbb{C})$	$\mathfrak{so}_{2n}(\mathbb{C})$	$\mathfrak{so}_{2p+1}(\mathbb{C})$	$p<q$
					$p+q=2n$
					quasi-split if $q=p+2$
DIII	$\mathfrak{so}^*(2n)$	$\mathfrak{so}_{2n}(\mathbb{C})$	$\mathfrak{so}_{2n}(\mathbb{C})$	$\mathfrak{sp}_p(\mathbb{C})$	$p=[n/2]$
EI	$\mathfrak{e}_{6(6)}$	$\mathfrak{e}_6(\mathbb{C})$	$\mathfrak{e}_6(\mathbb{C})$	$\mathfrak{e}_6(\mathbb{C})$	split
EII	$\mathfrak{e}_{6(2)}$	$\mathfrak{e}_6(\mathbb{C})$	$\mathfrak{e}_6(\mathbb{C})$	$\mathfrak{f}_4(\mathbb{C})$	quasi-split
EIII	$\mathfrak{e}_{6(-14)}$	$\mathfrak{e}_6(\mathbb{C})$	$\mathfrak{e}_6(\mathbb{C})$	$\mathfrak{so}_5(\mathbb{C})$	
EIV	$\mathfrak{e}_{6(-26)}$	$\mathfrak{e}_6(\mathbb{C})$	$\mathfrak{e}_6(\mathbb{C})$	$\mathfrak{sl}_3(\mathbb{C})$	
EV	$\mathfrak{e}_{7(7)}$	$\mathfrak{e}_7(\mathbb{C})$	$\mathfrak{e}_7(\mathbb{C})$	$\mathfrak{e}_7(\mathbb{C})$	split
EVI	$\mathfrak{e}_{7(-5)}$	$\mathfrak{e}_7(\mathbb{C})$	$\mathfrak{e}_7(\mathbb{C})$	$\mathfrak{f}_4(\mathbb{C})$	
EVII	$\mathfrak{e}_{7(-25)}$	$\mathfrak{e}_7(\mathbb{C})$	$\mathfrak{e}_7(\mathbb{C})$	$\mathfrak{sp}_3(\mathbb{C})$	
EVIII	$\mathfrak{e}_{8(8)}$	$\mathfrak{e}_8(\mathbb{C})$	$\mathfrak{e}_8(\mathbb{C})$	$\mathfrak{e}_8(\mathbb{C})$	split
EIX	$\mathfrak{e}_{8(-24)}$	$\mathfrak{e}_8(\mathbb{C})$	$\mathfrak{e}_8(\mathbb{C})$	$\mathfrak{f}_4(\mathbb{C})$	
FI	$\mathfrak{f}_{4(4)}$	$\mathfrak{f}_4(\mathbb{C})$	$\mathfrak{f}_4(\mathbb{C})$	$\mathfrak{f}_4(\mathbb{C})$	split
FII	$\mathfrak{f}_{4(-20)}$	$\mathfrak{f}_4(\mathbb{C})$	$\mathfrak{f}_4(\mathbb{C})$	$\mathfrak{sl}_2(\mathbb{C})$	
G	$\mathfrak{g}_{2(2)}$	$\mathfrak{g}_2(\mathbb{C})$	$\mathfrak{g}_2(\mathbb{C})$	$\mathfrak{g}_2(\mathbb{C})$	split

TABLE 1. Associated Lie algebras $\check{\mathfrak{h}}$ for non-compact real Lie algebras $\mathfrak{g}_\mathbb{R}$ with simple complexifications $\mathfrak{g}$. Notation following É. Cartan, and [Hel78].

THE NADLER GROUP

D.Nadler, *Perverse sheaves on real loop Grassmannians*, Invent. Math. **159** (2005) 1–73

- $G^r \subset G^c$

- $\Rightarrow \widehat{H}^c \subset {}^L G^c$

Conjecture: The mirror of the moduli space of flat G^r-bundles is supported on the Higgs bundle moduli space $\mathcal{M}(\widehat{H}^c) \subset \mathcal{M}({}^L G^c)$

- $SL(m, \mathbf{H}) \subset SL(2m, \mathbf{C})$...

- $SO(p, q) \subset SO(p+q, \mathbf{C})$

- spectral curve is non-reduced

- How does SYZ handle this degenerate fibre?

HYPERHOLOMORPHIC BUNDLES

- connection with curvature of type $(1,1)$ wrt I, J, K
- 4 dimensions = anti-self-dual
- $\Leftrightarrow$ holomorphic bundle on twistor space

- Levi-Civita connection is hyperholomorphic

- Higgs bundle tangent space $(\dot{A}, \dot{\Phi})$

- $\bar{\partial}_A \dot{\Phi} + [\dot{A}, \Phi] = 0$ modulo $(\dot{A}, \dot{\Phi}) = (\bar{\partial}_A \psi, [\psi, \Phi])$

- $=$ hypercohomology $\mathbb{H}^1$ of $\quad \mathcal{O}(\mathfrak{g}) \stackrel{[\Phi,-]}{\to} \mathcal{O}(\mathfrak{g} \otimes K)$

- holomorphic structure relative to I

- complex structure J: flat G^c-connection
- adjoint bundle $\mathfrak{g}$ with induced flat connection
- tangent space = de Rham cohomology $H^1(\Sigma, \mathfrak{g})$
- holomorphic structure relative to J

- any representation of G^c gives $\mathcal{O}(V) \xrightarrow{\Phi} \mathcal{O}(V \otimes K)$

- $\Omega^0(V) \to \Omega^{01}(V) \oplus \Omega^{10}(V) \to \Omega^{11}(V)$

  $\psi \mapsto (\bar{\partial}_A \psi, \Phi(\psi))$

- Hodge theory $\Rightarrow$ harmonic representative $=$

  solution to elliptic equation $\mathbf{D}^* \psi = 0$

- $\mathbf{D} : \Omega^0(V) \oplus \Omega^{11}(V) \to \Omega^{01}(V) \oplus \Omega^{10}(V)$

- Dirac operator

- $\mathbf{D}$ is formally quaternionic

- Higgs bundle equations $\Rightarrow \mathbf{D}\mathbf{D}^*$ is real

- $\Rightarrow$ coker $\mathbf{D}$ carries a hyperholomorphic connection

- "Dirac-Higgs bundle" (if a universal bundle on $\mathcal{M} \times \Sigma$ exists)

- representation $\rho : G \to U(m)$

- hypercohomology of associated bundle V

- defines a Dirac bundle $\mathbf{V}$ over $\mathcal{M}$

- ... take tensor products, exterior products of $\mathbf{V}$ etc.

- fixed Higgs bundle (U, Ψ) rank m and varying (V, Φ) rank n

- $(U \otimes V, \Psi \otimes 1 + 1 \otimes \Phi)$ rank mn Higgs bundle

- Dirac bundle over $\mathcal{M}(GL(mn, \mathbf{C}))$

- pull back to $\mathcal{M}(GL(n, \mathbf{C}))$ – hyperholomorphic

E.Franco & M.Jardim, *Mirror symmetry for Nahm branes*,
arXiv:1709.01314

VECTOR REPRESENTATION OF $GL(n, \mathbf{C})$

- $\mathcal{O}(V) \xrightarrow{\Phi} \mathcal{O}(V \otimes K)$

- $0 \to H^1(\ker \Phi) \to \mathbb{H}^1 \to H^0(\operatorname{coker} \Phi) \to 0$

VECTOR REPRESENTATION OF $GL(n,\mathbf{C})$

- $\mathcal{O}(V) \stackrel{\Phi}{\to} \mathcal{O}(V \otimes K)$

- $0 \to H^1(\ker \Phi) \to \mathbb{H}^1 \to H^0(\operatorname{coker} \Phi) \to 0$

- $\det \Phi = 0$ on $x = 0$ and $\operatorname{coker} \Phi \cong L$

  so $\quad \mathbb{H}^1 \cong \bigoplus_{x_i \in S \cap \{x=0\}} L_{x_i}$

- Dirac-Higgs bundle $\mathbf{V}$ hyperholomorphic

$U(m,m)$ REAL FORM

- mirror $\hat{\mathcal{M}} = Sp(2m, \mathbf{C})$-Higgs bundles

- $V = V_+ \oplus V_-$, $c_1(V_+) \in H^2(\Sigma, \mathbf{Z})$

- for a BBB-brane we need a hyperholomorphic bundle for each $c_1(V_+)$

- spectral curve $\det(x - \Phi) = x^{2m} + a_1 x^{2m-2} + \ldots + a_m$

- involution $\sigma(x) = -x$ on S

- $L = U\pi^* K^{(2m-1)/2}$, $U \in \mathrm{Jac}(S)$

- $\sigma^* U \cong U$

- $Y \cap A =$ fixed point set of $\sigma =$ abelian subvariety

- $\sigma^* U \cong U$

- action at fixed-point set ± 1

- $c_1(V_+) \sim$ (even) number of $+1's$

- $\{x_1, \ldots, x_\ell\} \subset S \cap \{x = 0\}$ defines a one-dimensional space

$$L_{x_1} \otimes L_{x_2} \otimes \cdots \otimes L_{x_\ell}$$

- vector space $\displaystyle\bigoplus_{\{x_1,\ldots,x_\ell\} \subset S \cap \{x=0\}} L_{x_1} \otimes L_{x_2} \otimes \cdots \otimes L_{x_\ell}$

- $\mathbf{V} = \bigoplus_{x_i \in S \cap \{x=0\}} L_{x_i}$

- $\Lambda^\ell \mathbf{V} = \bigoplus_{\{x_1,\ldots,x_\ell\} \subset S \cap \{x=0\}} L_{x_1} \otimes L_{x_2} \otimes \cdots \otimes L_{x_\ell}$

- induced hyperholomorphic connection

- $\mathbf{V} = \bigoplus_{x_i \in S \cap \{x=0\}} L_{x_i}$

- $\Lambda^\ell \mathbf{V} = \bigoplus_{\{x_1,\ldots,x_\ell\} \subset S \cap \{x=0\}} L_{x_1} \otimes L_{x_2} \otimes \cdots \otimes L_{x_\ell}$

- induced hyperholomorphic connection

- no universal bundle for $Sp(2m, \mathbf{C})$ ($\mathbf{Z}_2$-gerbe obstruction)

- ℓ even $\Rightarrow \Lambda^\ell \mathbf{V}$ well-defined

LAGRANGIAN CORRESPONDENCES

- $(M, \omega_M), (N, \omega_N)$ symplectic manifolds

- Lagrangian $Z \subset (M \times N, (\omega_M, -\omega_N))$

- e.g. graph of a symplectomorphism

- symplectic morphism (Weinstein)

EXAMPLE

- $M = N = T^*X$
- $Y \subset X$ submanifold
- $Z \subset T^*X \times T^*X = \{(x,\xi),(x,\eta) : (x,\xi-\eta) \in (TY)^0\}$

EXAMPLE

- $M = N = T^*X$
- $Y \subset X$ submanifold
- $Z \subset T^*X \times T^*X = \{(x,\xi),(x,\eta) : (x, \xi - \eta) \in (TY)^0\}$
- $\xi - \eta, \eta - \zeta \in (TY)^0 \Rightarrow \xi - \zeta \in (TY)^0$
- .... "idempotent" morphism

S-duality of boundary conditions and the Geometric Langlands program

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Abstract: Maximally supersymmetric gauge theory in four dimensions admits local boundary conditions which preserve half of the bulk supersymmetries. The S-duality of the bulk gauge theory can be extended in a natural fashion to act on such half-BPS boundary conditions. The purpose of this note is to explain the role these boundary conditions can play in the Geometric Langlands program. In particular, we describe how to obtain pairs of Geometric Langland dual objects from S-dual pairs of half-BPS boundary conditions.

- $X =$ moduli space of degree 0 stable $GL(n, \mathbf{C})$ bundles

- $Y = \{V : H^0(\Sigma, V \otimes K^{1/2}) \neq 0\}$ (determinant divisor)

- $\Rightarrow H^0(\Sigma, V^* \otimes K^{1/2}) \neq 0$ (RR + Serre duality)

- $s \in H^0(\Sigma, V \otimes K^{1/2}), \alpha \in H^0(\Sigma, V^* \otimes K^{1/2})$

  $\Rightarrow s \otimes \alpha \in H^0(\Sigma, \mathrm{End}\, V \otimes K)$

- normal bundle of Y = Quillen line bundle

- $L = (\Lambda^{top} \ker \bar{\partial}_A)^* \otimes (\Lambda^{top} \operatorname{coker} \bar{\partial}_A)$

- $s \otimes \alpha \in L^*$ = conormal bundle = $(TY)^0$

- normal bundle of Y = Quillen line bundle

- $L = (\Lambda^{top}\,\mathrm{ker}\,\bar{\partial}_A)^* \otimes (\Lambda^{top}\,\mathrm{coker}\,\bar{\partial}_A)$

- $s \otimes \alpha \in L^* =$ conormal bundle $= (TY)^0$

- Lagrangian relation in $\mathcal{M} \times \mathcal{M}$

$$\boxed{\Phi - \Phi' = s \otimes \alpha}$$

- $\Phi - \Phi' = s \otimes \alpha$

- eigenvectors: $\Phi^T \psi = \lambda \psi, \Phi' v = \lambda' v$

- $\langle \psi, s \rangle \langle \alpha, v \rangle = \langle \Phi^T \psi, v \rangle - \langle \psi, \Phi' v \rangle = (\lambda - \lambda') \langle \psi, v \rangle$

- $\Rightarrow \lambda = \lambda'$ if $\langle \alpha, v \rangle = 0$ or $\langle \psi, s \rangle = 0$

- $\lambda = \lambda'$

- resultant of $\det(x - \Phi), \det(x - \Phi')$

- $n^2(g-1)$ points in Σ

- $\lambda = \lambda'$

- resultant of $\det(x - \Phi), \det(x - \Phi')$

- $n^2(g-1)$ points in Σ

- $S \cap S' = D + D'$

  $\langle \alpha, v \rangle = 0$ or $\langle \psi, s \rangle = 0$

- two spectral curves S, S'

- $S \cap S' = D + D^*$ $\quad (\lambda_i = \lambda_i')$

- D divisor of t, D^* of t^*

- $D \cong L\pi^* K^{1/2}$ on S and $L'\pi^* K^{1/2}$ on S'

CORRESPONDENCE

- spectral curve plus $(g_S - 1)$ points $\sim D$

- effective divisor $D^* \sim K_S(-D)$

- $D + D^* \in H^0(S, K_S) = H^0(S, \pi^* K^n)$

- pencil of spectral curves S' through $D + D^*$

THE MIRROR BBB BRANE

- ${}^{L}GL(n, \mathbf{C}) = GL(n, \mathbf{C})$

- $Z \subset \mathcal{M} \times \mathcal{M} \to B \times B$ surjective

- fibre $(g_S - 1)$-element subsets $U \subset S \cap S'$

- $\mathbf{V} = \bigoplus_{x \in S \cap S'} L_x$

- $\mathbf{V}$ hyperholomorphic?

- $GL(n, \mathbf{C}) \times GL(n, \mathbf{C})$ acting on $\mathbf{C}^n \otimes \mathbf{C}^n$

- $\Phi \otimes 1 - 1 \otimes \Phi' : \mathcal{O}(V_1 \otimes V_2) \to \mathcal{O}(V_1 \otimes V_2 \otimes K)$

- determinant of $X \mapsto AX - XB =$

  resultant of $\det(x - A), \det(x - B)$

- $\mathbb{H}^1$ supported where $\lambda = \lambda'$

- $\mathbf{V} = \bigoplus_{x \in S \cap S'} L_x$ hyperholomorphic

- fibre $\sim$ $(g_S - 1)$-element subsets

- basis $\{e_1, \ldots, e_n\}$ of vector space U

- basis for $\Lambda^m U \sim m$-element subsets

- mirror of $Z = \Lambda^{g_S - 1} \mathbf{V}$

- no universal bundle

- $\mathbf{V}_\alpha$ defined on open set U_α

- $\mathbf{V}_\alpha \cong L_{\alpha\beta} \otimes \mathbf{V}_\beta$ on $U_\alpha \cap U_\beta$

- $L^n_{\alpha\beta}$ trivial ($\mathbf{Z}_n$-gerbe)

- $(g_S - 1) = n^2(g-1)$ so $\Lambda^{g_S-1}\mathbf{V}$ well-defined.